

# Dynamic movement primitives for movement generation motivated by convergent force fields in frog

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Dynamic movement primitives (DMP) have been introduced to humanoid robotics as a general framework for motion generation [1, 2]. They represent a motion with a set of differential equations. These equations can be adapted to generate any movement trajectory. Among the favorable properties are a simple generalization of a represented movement to new targets (just by changing a goal parameter) and robustness against perturbations due to the attractor dynamics of the equations. Here, we demonstrate a link between DMP and neurophysiology: DMP can be motivated from convergent force fields that are observed in the frog after spinal-cord stimulation [3]. Moreover, this derivation provides us with a new form of DMP that generalize better towards new goals.

In the following, we describe how DMP are used to generate motion, show the motivation and derivation from neurophysiology, and finally, demonstrate the utility of the new equations in a robot experiment with the Sarcos slave arm.

## Dynamic movement primitives

A dynamic movement primitive generates a motion  $\mathbf{x}(t)$  – e.g., of an end-effector – with a differential equation [1]. This equation is built from a damped spring around a goal position  $\mathbf{g}$  and a non-linear perturbing acceleration (here, written in vector form):

$$\dot{\mathbf{v}} = \mathbf{K}(\mathbf{g} - \mathbf{x}) - \mathbf{D}\mathbf{v} + \mathbf{F}(\mathbf{W}, \theta)(\mathbf{g} - \mathbf{x}_0) \quad (1)$$

$$\dot{\mathbf{x}} = \mathbf{v} , \quad (2)$$

where  $\mathbf{x}_0$  is the start position of a movement,  $\mathbf{K}$  the spring constant,  $\mathbf{D}$  the damping constant, and  $\mathbf{F}$  a diagonal matrix whose elements  $F_{kk}$  contain the parametrized non-linear functions

$$F_{kk} = \frac{\sum_i \psi_i(\theta) w_{ik} \theta}{\sum_i \psi_i(\theta)} , \quad (3)$$

which are normalized sums of Gaussian functions  $\psi_i(\theta)$ . The non-linear functions depend on a phase variable  $\theta$ , which runs from 1 towards 0 according to

$$\dot{\theta} = -\alpha \theta , \quad (4)$$

where  $\alpha$  is a predefined constant. The centers of  $\psi_i(\theta)$  are fixed and distributed between 0 and 1.

The weights  $w_{ik}$  are adapted to a desired trajectory: first, a movement  $\mathbf{x}(t)$  is recorded and its derivatives  $\mathbf{v}(t)$  and  $\dot{\mathbf{v}}(t)$  are computed for each time step  $t$ . Second,  $\mathbf{F}(t)$  is computed based on (1). Third, (4) is integrated and  $\theta(t)$  evaluated. Using these arrays, we find the weights  $w_{ik}$

in (3) by linear regression, which can be solved efficiently.

## Motivation from neurophysiology

To derive our framework for motion generation, we use three key neurophysiological findings in frog [3]:

- After stimulating the spinal cord, a force field can be observed by measuring forces at different leg positions. These fields are often convergent.
- The magnitude of force fields is modulated in time by bell-shaped time pulses.
- Simultaneously stimulated force fields add up linearly.

These findings are realized in the model as follows. We make a first-order approximation of a convergent field around  $\mathbf{w}_i$ ,

$$\chi_i(\mathbf{x}, \mathbf{v}) = \mathbf{K}(\mathbf{w}_i - \mathbf{x}) - \mathbf{D}\mathbf{v} . \quad (5)$$

Each field is modulated over time with a Gaussian function centered at time  $c_i$ ,

$$\psi_i(t) = \exp(-h(t - c_i)^2) . \quad (6)$$

We use the summation property to obtain a more complex field,

$$\varphi(\mathbf{x}, \mathbf{v}, t) = \frac{\sum_i \psi_i(t) \chi_i(\mathbf{x}, \mathbf{v})}{\sum_i \psi_i(t)} . \quad (7)$$

Different from the force fields in frog, here, we use acceleration fields, i.e.,  $\dot{\mathbf{v}} = \varphi(\mathbf{x}, \mathbf{v}, t)$ . In our robot application, we use inverse kinematics and dynamics to compute the joint torques.

Combining (5) with (7) results in the equations of motion

$$\dot{\mathbf{v}} = \mathbf{K} \left( \frac{\sum_i \psi_i(t) \mathbf{w}_i}{\sum_i \psi_i(t)} - \mathbf{x} \right) - \mathbf{D}\mathbf{v} \quad (8)$$

$$\dot{\mathbf{x}} = \mathbf{v} . \quad (9)$$

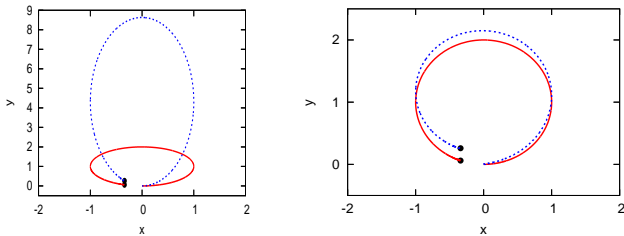
To make the equation of motion converge to the goal  $\mathbf{g}$ , we add around  $\mathbf{g}$  another field (5) and shift the weight from (8) to the new field. As weight, we use the phase variable  $\theta$ , as computed by (4), and thus, the new acceleration field becomes

$$\begin{aligned} \dot{\mathbf{v}} = & \theta \mathbf{K} \left( \frac{\sum_i \psi_i(\theta) \mathbf{w}_i}{\sum_i \psi_i(\theta)} + \mathbf{x}_0 - \mathbf{x} \right) \\ & + (1 - \theta) \mathbf{K}(\mathbf{g} - \mathbf{x}) - \mathbf{D}\mathbf{v} . \end{aligned} \quad (10)$$

We inserted an extra  $\mathbf{x}_0$  to make the equation translation invariant. Furthermore, we changed the dependence of  $\psi$  on  $t$  to  $\theta$ . As in the original DMP, this change will allow more flexibility since we can manipulate  $\theta$ ; absolute timing cannot be easily modified. Equation (10) can be rewritten into

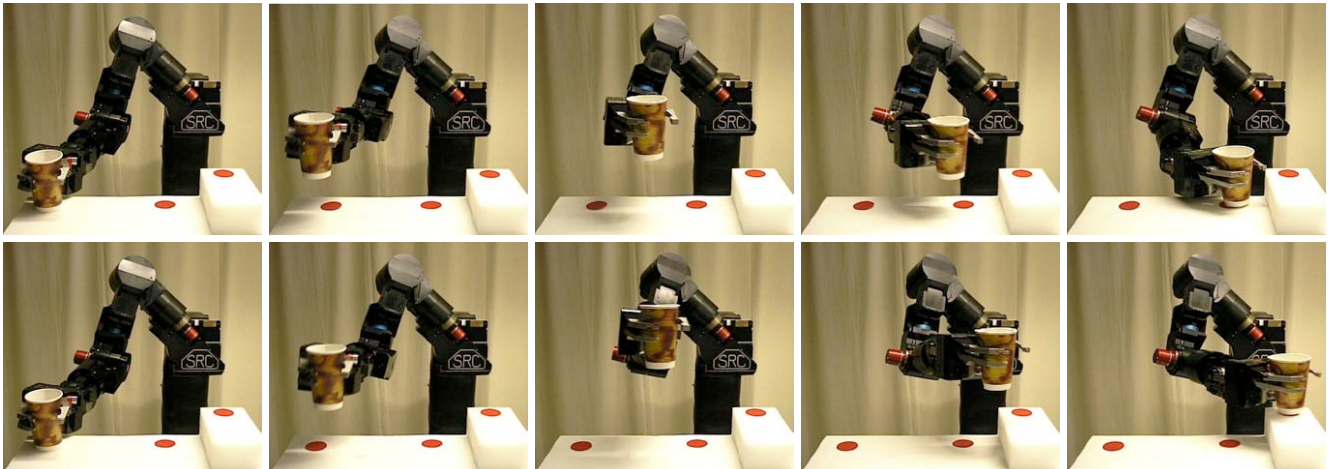
$$\dot{\mathbf{v}} = \mathbf{K}(\mathbf{g} - \mathbf{x}) - \mathbf{D}\mathbf{v} - \mathbf{K}(\mathbf{g} - \mathbf{x}_0)\theta + \mathbf{K}\mathbf{f} , \quad (11)$$

where  $f_k = F_{kk}$ . This equation is similar to the original form of DMP - see (1). In the limit  $\theta \rightarrow 0$ , both forms are the same. The main difference is that the non-linear term does not scale with  $(\mathbf{g} - \mathbf{x}_0)$ . This avoids problems that occur if  $\mathbf{g}$  and  $\mathbf{x}_0$  are close in any dimension (see Fig. 1). Our robot demonstration - in which the height of the end-effector at start and end point of a movement is the same - works with the new formulation, but not with the original one. To learn a movement from demonstration, we again adapt  $\mathbf{w}_i$  using the same technique as for the original DMP.



**Fig. 1:** Comparison of goal adaptation between original (Left) and new (Right) DMP formulation for a 2-dimensional movement in  $(x, y)$ . The same demonstrated movement (solid curve) and goals (black dots) are used for both formulations. The dashed curves show the results of changing the goal  $\mathbf{g}$  for the entire movement (without changing  $w_{ik}$ ).

## Robot demonstration



**Fig. 2:** Placing a cup with the Sarcos slave arm. The first row shows the reproduction of a demonstrated movement. The second row shows the generalization to a new goal position.

Our robot experiment demonstrates movement reproduction and generalization to a new goal position using the new biologically-motivated DMP formulation. The robot places a cup between two positions on a table. The placing movement was pre-recorded from a human demonstrator. Only the end-effector position  $\mathbf{x}(t)$  was recorded. Afterwards, the movement equations were adapted to reproduce this movement by computing the appropriate  $\mathbf{w}_i$  parameters. Given  $\mathbf{x}(t)$ , we computed the joint torques of the robot using the known inverse kinematics and dynamics. The robot could reproduce the demonstrated movement and generalize to a new goal position simply through changing the goal position  $\mathbf{g}$  of the placing movement (Fig. 2).

## Conclusions

We motivated the dynamic-movement-primitive framework from convergent force fields observed in frog. The neurophysiologically-motivated derivation resulted in a new form of differential equations for motion generation that exhibit a more robust and human-like generalization of a represented movement to new targets. The utility of these equations was demonstrated in robot experiments.

## References

- [1] A. J. Ijspeert, J. Nakanishi, and S. Schaal, “Learning attractor landscapes for learning motor primitives.” in *Advances in Neural Information Processing Systems*, S. Becker, S. Thrun, and K. Obermayer, Eds., vol. 15. MIT Press, Cambridge, MA, 2003, pp. 1523–1530.
- [2] S. Schaal, “Dynamic movement primitives: A framework for motor control in humans and humanoid robotics,” in *2nd International Symposium on Adaptive Motion of Animals and Machines (AMAM)*, 2003.
- [3] S. F. Giszter, F. A. Mussa-Ivaldi, and E. Bizzi, “Convergent force fields organized in the frog’s spinal cord,” *Journal of Neuroscience*, vol. 13, no. 2, pp. 467–491, 1993.