INSTITUTE OF PHYSICS PUBLISHING CLASSICAL AND QUANTUM GRAVITY

Class. Quantum Grav. **19** (2002) 1709–1716 PII: S0264-9381(02)30046-7

Cross-correlation studies with seismic noise

H Hoffmann, J Winterflood, Y Cheng and D G Blair

Department of Physics, University of Western Australia, Perth 6009, Australia

Received 25 October 2001
Published 14 March 2002
Online at stacks.iop.org/CQG/19/1709

Abstract
Ocean waves interacting in shallow water at the shore generate land waves propagating inland. To study these waves vertical, horizontal and tilt seismic noise were measured simultaneously at one location. Vibration isolators designed for gravitational wave research were used for detection. Cross-correlation between the above components was calculated. We found correlations between all of them. However, only the correlation between horizontal and vertical motions could be addressed to land waves, and other correlations are thought to be due to local rigid body motion of the large building in which the experiments were located.

PACS number: 0480N

1. Introduction

Our motivation for studying cross-correlation on seismic noise was twofold. First, the isolation of seismic noise is a huge problem in the design and construction of gravitational wave detectors (see [1] and references therein). In particular, gravity gradient noise forms a fundamental limit to the sensitivity obtainable at low frequencies. Therefore, it is desirable to understand more about the nature of this seismic noise as, with sufficient measurement, it may even be possible to reduce this noise source by predicting and subtracting some of its effects from the data. Second, so far, only a few experimental studies have been done on Rayleigh waves (R-waves, sometimes called ground roll waves) as induced by ocean waves. This paper concentrates on the measurement of these surface waves.

R-waves can be generated by an oscillating pressure on the ground surface [2]. Two thirds of the generated energy is imparted into R-waves and the other third in shear and compression waves. We assume that the varying pressure of the incoming ocean waves, as they enter shallow water, provides the driving pressure for their generation. Figure 1 illustrates the above process for ocean waves. This is borne out by the remarkable correlation between microseismic signals in the 0.04–0.25 Hz band demonstrated by Liu [3] and Woo [4]. Figure 2 is an example of this correlation [4]. The resulting waves propagate in land as a surface wave. We will call this kind of wave ‘land waves’. Little is known about their speed (we assume it is around 1 km s^{-1}). In R-waves, the surface-particle motion (relative to a rest frame) describes

 © 2002 IOP Publishing Ltd Printed in the UK

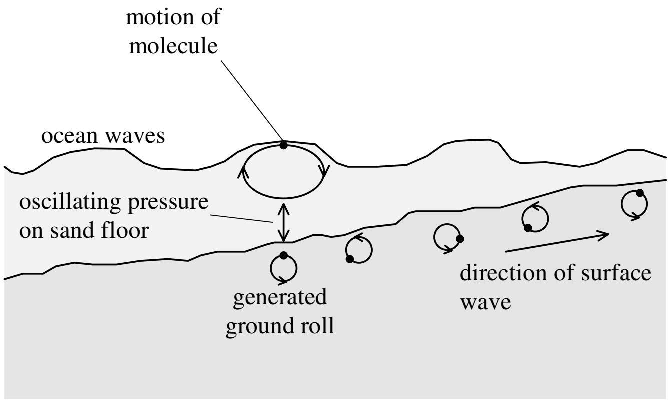

Figure 1. The oscillating pressure on the sea bed induces a surface wave which propagates inland.

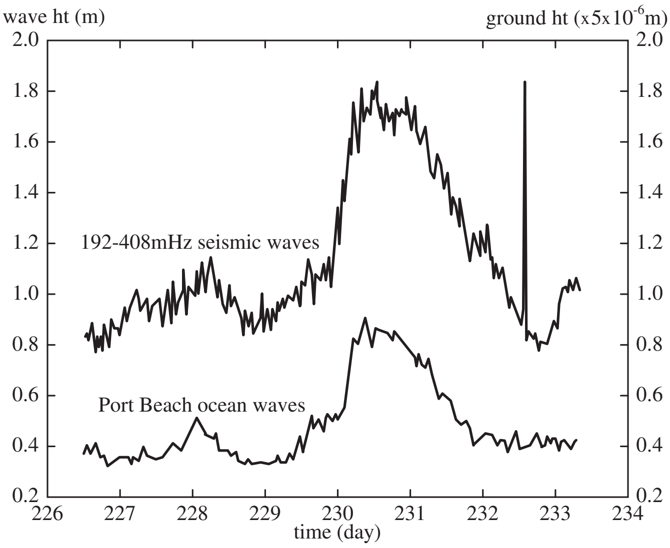

Figure 2. Port Beach wave height (14–21 August, 9 km south-west of UWA) and microseisms at UWA.

a retrograde ellipse. The plane of the ellipse points in the direction of wave propagation, and it is perpendicular to the surface of the ground. For a homogeneous medium, the ratio between vertical and horizontal components is unity at the surface and above unity below the surface [2]. We measured horizontal and vertical motions to identify ground roll.

For a Fourier component assuming that the size of the above ellipse is much smaller than the wavelength, the vertical excitation (v) can be described by a standard wave equation

$$v(x,t) = v_0 \sin(kx - \omega t). \tag{1}$$

This results in the following tilt θ motion:

$$\theta(x,t) = v_0 k \cos(kx - \omega t) = \theta_0 \cos(kx - \omega t). \tag{2}$$

Table 1. The table shows the characteristics of the three vibration isolation systems used to measure the three components (of interest) of seismic noise.

	LaCoste (vertical)	Scott–Russel (horizontal)	Tilt sensor
Period (s)	15.5	13	17
Q-factor	4.5	4	10
Suspended mass (kg)	370	2.53	50

There is a $\pi/2$ phase shift between tilt and vertical motion. If we were able to detect this phase shift, the above relation would allow us to determine the speed c (equation (3)) of the land waves by measuring vertical and tilt motions at the same position.

$$c = 2\pi f\, v_0/\theta_0. \tag{3}$$

An alternative method of measuring the tilt would be to measure the wave travel time between two reasonably separated points. In this case one needs only to measure a single component of the wave. In this paper we will present methods for the correlation of v and θ at one point.

2. Methods

We measured the vertical, horizontal and tilt components of seismic motion with three independent suspension systems. For the vertical motion, a LaCoste [5] pre-isolator stage was used, as designed for the pre-isolation system [6] for the Australian International Gravitational Observatory. This pre-isolator was operated in a vacuum tank (10^{-1} Torr) and its characteristics are shown in table 1. In principle a LaCoste isolator is equivalent to a simple spring mass system. The vertical position was read out with a shadow sensor (dynamic range 10 mm) and it was servo controlled to the centre of its read-out with a simple integral circuit with a very long time constant ($RC > 1000$ s). The output was fed back as a current in the suspending springs providing control of the vertical position by thermal weakening of the modulus of the springs—counteracting the springs' intrinsic temperature sensitivity. The large time constant was obtained with the aid of a 1 F supercap.

The horizontal motion was measured with a Scott–Russel pendulum [7], which is equivalent to a very long simple pendulum (here 42 m). A tubular design was used similar to one leg of figure 5 of [7] with a simple weight on the top and using the taut wire vertical motion constraint at the bottom of the beam as in figure 2 of [7]. The whole movable structure was placed in an aluminium tube which functioned as the basic frame and protected the structure from air movements. The position was read out with a two-dimensional (2D) shadow sensor with a dynamic range of 1 mm. Both north–south and east–west directions were recorded. Very long time-constant analogue integrators were also used to stabilize the Scott–Russel pendulum since it has a tendency to drift. The servo actuation was applied by coils, one for each axis, with a small permanent magnet for each coil mounted on the suspended mass. A Scott–Russel pendulum does not purely measure the horizontal vibration. Pendulum displacement occurs as a result of a combination of tilt and horizontal motions. But, as will be seen from the resulting spectra, the tilt component was small enough to be neglected.

The tilt was measured using a double flexure inverse pendulum [8–10]. A massive disc (50 kg) was suspended from a rod with x–y flexures at each end. The disc is attached to the rod by a 2D flexure with the centre of rotation slightly below its centre of mass. This provides some inverse pendulum or anti-spring to counteract the spring-rate of the flexure and results in a low Q-factor for the tilt mode. Another flexure connects the top of the rod to the frame.

This gives a relatively high frequency (~2 Hz) and high Q-factor pendulum mode which is damped by magnetic eddy current damping. (The orientation of the magnets and plates is arranged mainly to damp the pendulum mode, and also to damp the tilt mode a little.) The displacement of the disc's rim due to tilt was read out with shadow sensors, one for each direction (north–south and east–west). Thermal drifts are an issue, but no integrator feedback was required for equilibrium position control in this case due to good thermal stability during the period of the measurement. Table 1 shows the characteristics of all three devices.

All three devices were located in the basement of the Department of Physics (on the Crawley Campus, Perth). Vertical, horizontal and tilt motions were recorded simultaneously using LabView on a PC equipped with a Lab-PC+ (National Instruments) data acquisition board. The recording started on Friday, 29 June 2001 at 6.45 pm and lasted for 9.8 h, sampling at 10 Hz. Probably, no person entered the basement during that time.

3. Seismic noise spectra

From the obtained data, seismic noise spectra were calculated. The whole data array was split into files of length 4096 samples (corresponding to 409.6 s). On each of the files an FFT was performed and then the absolute values of all FFTs were averaged over all files. The resulting spectra were smoothed by averaging over a window of five adjoining points.

The transfer function T_f (detected vibration versus real vibration as a function of frequency f) for a simple pendulum or spring mass system is readily shown to be

$$T_f(f) = \left\| \frac{f^2 Q}{\left(f^2 - f_0^2\right) Q - \mathrm{i} f_0 f} \right\| \tag{4}$$

where Q is the quality factor and f_0 is the resonance frequency of the isolator. Both spectra of vertical and horizontal motions were corrected with the above transfer function using the values for Q and f_0 from table 1.

4. Cross-correlation

On the same data, cross-correlations between the different vibration measurements were calculated. Equation (5) defines the correlation function used:

$$\mathrm{Corr}(\tau) = \frac{\sum_n a(t_n) b(t_n + \tau)}{\sqrt{\sum a(t_n)^2 \sum b(t_n)^2}} \tag{5}$$

where $a(t_n)$ and $b(t_n)$ are the two data arrays, τ is the time shift between the two arrays. As above, the data were split into files of 4096 samples, and cross-correlations were averaged over these files. Before calculating cross-correlations the signals were frequency filtered, using a 0.04–0.2 Hz band. This frequency range was chosen as it had been found by an extensive trial and error search [4] to give the best correlation between ocean wave height and ground motion. The band was left reasonably broad to avoid forcing the signal into a harmony.

5. Results

The resulting seismic spectra are shown in figure 3(*a*). The spectra suggest a connection between vertical and horizontal vibrations. Between 0.1 and 0.2 Hz, the vertical and horizontal displacement curves show similar characteristics. For two frequencies (0.095 and 0.145 Hz), both vertical and horizontal curves show maxima.

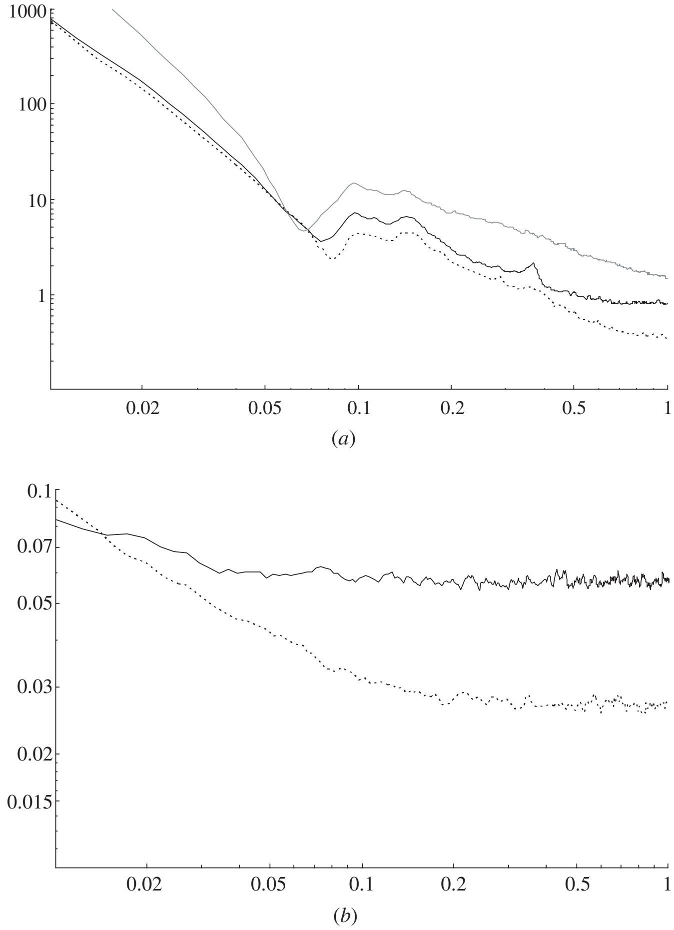

Figure 3. (*a*) Measured seismic displacement in μm Hz$^{-1/2}$ (solid: north–south, dashed: east–west, grey: vertical), frequency in Hz; (*b*) measured seismic tilt in μrad Hz$^{-1/2}$ (solid: north–south, dashed east–west), frequency in Hz.

In figure 3(*b*) the tilt noise spectrum is shown and aside from the N–S/E–W difference in level (possibly due to the shape and attachment of the concrete slab of the floor) it is remarkably featureless. This measured tilt spectrum is significantly above the noise floor measured with a clamped sensor [10] and has not been corrected for transfer function variation. The transfer function was measured [10] by applying known tilts using piezo actuators in the legs and gave values of around 1 ($\pm$20%) for frequencies above 0.2 Hz, but rose to 10 ($\pm$10%) around 0.05 Hz, suggesting that the tilt spectra in figure 3(*b*) at this frequency are higher than actual. Below this frequency the measured transfer function did not match the calculated one and so the tilt spectrum below 0.05 Hz may be suspect. (For more detail on the tilt sensor, see [10].) From a simplistic sinusoidal surface wave argument, and assuming constant wave velocity, one would expect the tilt spectrum to be similar in shape to the vertical spectrum in figure 3(*a*) but with a slope increase of +1. In this case the slope seems about right but the expected features are absent.

From the obtained spectra it could be deduced that the Scott–Russel unit measured mostly horizontal motion. At 0.1 Hz the measured tilt noise per $\sqrt{\text{Hz}}$ was 6×10^{-8} rad (figure 3(*b*),

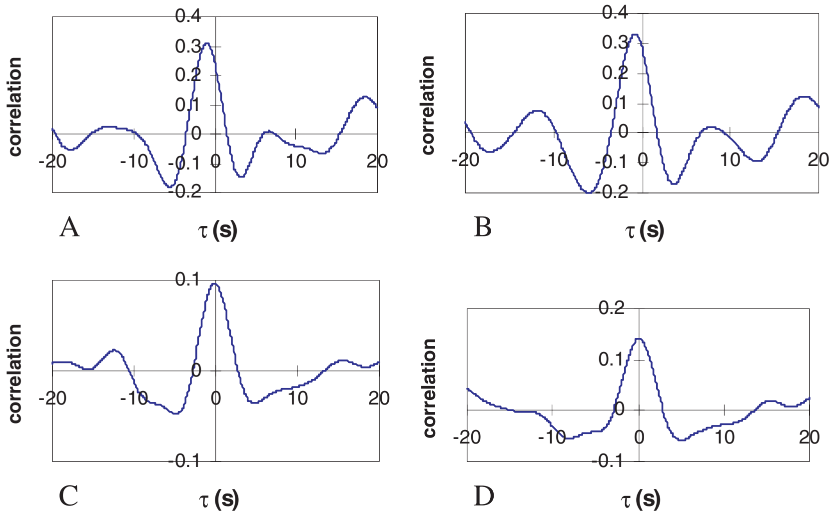

Figure 4. Cross-correlation results of seismic noise between (*a*) vertical and north–south horizontal, (*b*) vertical and east–west horizontal, (*c*) vertical and north–south tilt and (*d*) vertical and east–west tilt. Here τ is the time shift between captured signals. A time delay of 1.0 s was observed for horizontal versus vertical. However, negligible phase shift was observed between vertical and tilt noise. Before calculating correlations the signal was frequency filtered using a 0.04–0.2 Hz band.

north–south) which results in a displacement on the Scott–Russel of 3.5 μm (this was calculated by assuming that the isolator was equivalent to a simple pendulum of 42 m length). The value 3.5 μm has to be compared with the measured 8 μm displacement (figure 3(*a*), north–south). For higher frequencies this relation shifts towards a lower tilt contamination because the tilt transfer function drops as $1/f^2$ while the T_f for horizontal motion approaches a constant value, 1 (see equation (4)). Thus tilt contamination of the Scott–Russel is not serious.

Figure 4 shows the correlation between the seismic vertical, horizontal and tilt motions. The resulting correlations showed a time delay of 1.0 s between the horizontal and vertical motions (figures 4(*a*) and (*b*)). However, there was no time shift between vertical and tilt motions (figures 4(*c*) and (*d*)). Had there been larger time shifts, it would have been useful to have filtered the data into narrower bands to obtain phase shifts rather than time, but with the small times found it was simpler and more effective not to spread out the data more. There was no significant difference observed between the north–south and east–west directions.

6. Discussion

The time delay (of 1 s) between the vertical and horizontal vibrations and the corresponding spectra suggest the detection of ground roll, possibly, as from land waves. However, the phase shift represented by this time delay is much smaller than the expected $\pi/2$ for ground roll (period of land waves: 7–10 s). A possible explanation might be that surface particles move (relative to a rest frame) along an elliptic rather than a circular curve (which is also suggested by the difference in amplitude between vertical and horizontal, see figure 3(*a*)) and that this ellipse is tilted by some angle. This is illustrated in figure 5(*a*). This motion can be regarded as a generalization of the circular ground roll. The horizontal displacement is larger for the north–south direction than for the east–west direction (figure 3(*a*)). This would indicate a

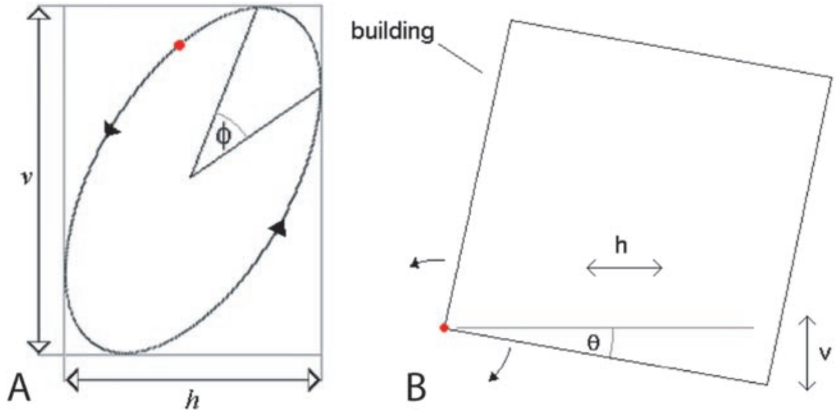

Figure 5. In (*a*), the motion of a ground particle relative to a rest frame is shown as expected for a land surface wave. The tilt of the ellipse is a generalization, which allows the phase shift ϕ between vertical (v) and horizontal (h) motions to be less than a quarter of the wave period (as observed). In (*b*), a possible explanation is given for the zero phase shift between vertical and tilt θ. The building in which the experiment was located vibrates as a whole, e.g. due to wind.

direction of wave propagation closer to north–south than east–west (while the shore is towards west of Crawley). It is unclear if this is possible.

The negligible phase shift between vertical and tilt differs significantly from the expected $\pi/2$ phase shift for land waves (see equation (2)). The measured tilt floor is higher than the expected tilt value due to a land wave. Assuming a wavelength of 6 km and a vertical displacement of 10 μm Hz$^{-1/2}$ (figure 3(*a*)), the result is a tilt of 10^{-8} rad Hz$^{-1/2}$. This value is significantly lower than the measured 3 to 6×10^{-8} rad Hz$^{-1/2}$ (figure 3(*b*)). Thus, this suggests that some other source is responsible for the observed correlation. Figure 5(*b*) gives a possible explanation. The building containing the experiment could be shaking as a whole (e.g. as a result of wind). This movement has zero phase shift between tilt and vertical motions.

We could not determine the speed of the land waves suggested by equation (3). A more promising method seems to be the study of cross-correlation with horizontal excitations measured at distant locations. As an alternative, one might perform an equivalent experiment outside the building.

Acknowledgments

The first author (HH) wishes to thank all the members of the gravity wave research group at the UWA for their help and collaboration, and especially thanks D G Blair for his constructive ideas and for providing the basis of all the research done in the group. This study was supported by the Australian Research Council, and is part of the research programme of the Australian Consortium for Interferometric Gravitational Astronomy.

References

[1] Ju L, Blair D G and Zhao C 2000 *Rep. Prog. Phys.* **63** 1317
[2] Richart F E, Hall J R and Woods R D 1970 *Vibrations of Soils and Foundations* (Englewood Cliffs, NJ: Prentice-Hall) ch 3
[3] Liu J F, Blair D G and Ju L 1998 *Meas. Sci. Technol.* **9** 1772

[4] Woo L M 1998 Wave measurements using a micro-seismometer *UWA Honours Thesis*
[5] LaCoste L 1983 *Geophysics* **48** 606
[6] Winterflood J, Zhou Z B and Blair D G 2000 Ultra-low residual motion suspension system *Proc. 2nd TAMA Int. Workshop on Gravitational Wave Detection (National Olympics Memorial Youth Center, Tokyo, Japan, 19–22 Oct. 1999) Gravitational Wave Detection II* (Tokyo: Universal Academy) pp 301–10
[7] Winterflood J and Blair D G 1996 *Phys. Lett.* A **222** 141
[8] Winterflood J, Zhou Z B, Ju L and Blair D G 2000 *Phys. Lett.* A **277** 143
[9] Zhou Z B, Winterflood J, Ju L and Blair D G 2001 *Phys. Lett.* A **280** 197
[10] Cheng Y, Winterflood J, Ju L and Blair D G 2002 Tilt sensor and servo control system for gravitational wave detection *Proc. of the 4th Edoardo Amaldi Conf. on Gravitational Waves (Perth, Western Australia, 8–13 July 2001) Class. Quantum Grav.* **19** 1723