Toward Ultra High Speed Locomotors: Design and Test of a Cheetah Robot Hind Limb

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***Abstract*—High speed robot locomotion is one of the most challenging problems in mobile robots. Fast robots push the limits of mechanical design, control and perception. The cheetah is an existence proof of what the authors term an Ultra-High Speed Locomotor, which can attain speeds of greater than 50 leg lengths per second and can cover 10 meters in a single gait cycle.**

In this paper, the energetics and dynamics of high-speed quadruped movement are analyzed. A leg design is presented that combines a novel hybrid actuator concept plus biarticular muscles to create a lightweight (17 Newton) leg capable of generating 90 Newtons of force.

Static and dynamic test results show the promise of the hybrid actuator system in ultra high-speed locomotors.

I. Introduction

High speed running or galloping is of great interest to the robotics community as it places extreme demands on our understanding of mechanical design, perception, and control. The mechanical design must be capable of the extreme velocity and force needed to achieve a flight phase [1]. Anticipatory sensing of the environment, using vision, involves the tight integration of vision and perception to achieve seamless running. Control takes advantage of the stabilizing properties of the biomechanics and judicious use of proprioceptive and exteroceptive sensory channels. Finally, construction of a robot that mimics the biomechanics of a real animal will help us better understand the complex interplay between neural control and biomechanics.

Nature sets the performance bar for high-speed locomotion very high. It is in the animal kingdom where we see an existence proof of what we term *Ultra High-Speed Locomotors*. The cheetah is the fastest land animal capable of speeds in excess of 32 m/s. Assuming a leg length of .6 meters, the cheetah has a normalized speed of more than 50 leg lengths per second The running frequency of a cheetah is about 3 Hertz; with each gait cycle, the animal covers an impressive 10 meters.

To date the fastest robot runners have used prismatic joints. The current land speed record of a legged robot is 5.8 meters/sec at more than 9 leg legs per second set by the MIT leg lab [2], [1]. The closest contender is probably Sprawlita with a speed of about 8 leg lengths per second at Stanford [3]. In both cases, the robots used prismatic legs, and in both cases pressurized gas was used as the main source of energy.

The force and velocity requirements needed for running are difficult to achieve in an articulated limb. Robots with articulated limbs have been slower than robots with prismatic limbs. In robots with articulated limbs, the fastest claimed walking is a little over 3.5 leg lengths per second by Geng and colleagues [4] [1]. Other notable articulated limbs include the MIT Uniroo which has the distinction of being an articulated leg that also has a sustained flight phase. The MIT lab's Uniroo robot moved at an estimated 1.4 leg lengths per second [6], but it is not clear if its running speed had been optimized, i.e. it was likely capable of a higher top speed. Other work in improving the performance of articulated limbs for high speed locomotion include the Stanford/Ohio State galloping quadruped [7] and work by Seyfarth [8], [9], [10]. The Asimo Robot was capable of a modest flight phase as well.

An articulated leg has advantages over a prismatic leg. First, an articulated leg can have greater ground clearance than a simple telescoping tube. Second, as the configuration of the limb changes, torque loading on each joint changes, which can be used to the robot's advantage. When a robot walks or stands with a straight knee configuration, joint torques are minimized as the leg Jacobian tends toward singularity. Being close to the leg singularity can improve walking efficiency by minimizing energy needed for gravity compensation. Third, as Ruina and colleagues have demonstrated [11], [12] under strict conditions, the articulated limbs of bipeds require only a small amount of energy to walk with natural movement. It is reasonable to expect that an articulated limb will also be somewhat more compatible with the dynamics of trotting and galloping than a prismatic limbed robot.

Beyond kinematics, the biomechanics of articulated limbs are complex and not completely understood. It is suspected that special "biarticular" muscles, which operate on two joints, may play a critical role. Biarticular[2] means that a muscle spans two articulations (joints). These muscles are bifunctional as well. This means that with contraction they may cause one joint to flex (contract) while another joint is made to

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[1]RedBot achieved a similar speed using a similar robotic mechanism [5] although the walking rate was not reported directly in that work.

[2]In earlier papers we used the term "biarticulate" following Winter, however, we now use the more frequently used term "biarticular."

extend. Biarticular muscles are thought to transmit power from hip to knee to ankle during explosive movements such as jumping and running and to distribute force during landing [13]. In essence, biarticular muscles form a flexible energy transmission system within the limb. This is a capability which has not been duplicated in a prismatic limb.

Like prismatic legs, articulated legs can behave as springs [8], [9], [10], [14]. The elastomeric properties of limb tendons and muscles are important dynamic components of locomotion. Elastic elements reduce shock and can help thrust the animal or robot forward during the second half of stance phase. In dog galloping, the raw forces generated by muscles begin to dominate [15] and elastomeric effects become proportionately less.

In Section II we give a concise, general review of the energetics of animal locomotion and compute the theoretical power the ground reaction forces, the individual joint velocities and extents of movement, and the energy used at each joint needed for a quadruped cheetah robot to gallop.

In Section III we give specific information about the cheetahs kinematics and dynamics. In Section IV we give an overview of a robotic "cheetah" hind limb design. Section V gives preliminary performance results. Finally, Section VI is the conclusion and future work.

II. Energetics of Galloping

Several factors will determine the running efficiency of robots and animals: (1) The efficiency of the mechanical structure in transforming mechanical work into forward movement; (2) The efficiency of the actuator system in converting stored energy to mechanical work; (3) Environmental effects including soil characteristics, terrain, slope and aerodynamic effects; (4) energy storage and recovery.

Energetics has been studied in a wide range of animals [16], [17], [18], [19], [20] as well as robots [21], [22], [12], [23]. Through animal studies, remarkable conclusions have been drawn: the energy cost of running per unit distance per kilogram is to a first order independent of speed and type of animal, or how it uses its legs. The weight of the animal dictates its locomotory efficiency with larger animals being more efficient than smaller animals. The key determining factors of energy usage are weight and distance traveled. The cost of moving a given distance is relatively constant once a gait pattern has been selected.

Drawing data from Taylor [20], a cheetah uses 6 mL of oxygen per gram of body weight per hour to run at 30 km/h. Knowing that twenty joules of energy is liberated for every 1 mL of oxygen consumed, we can compute that each kg of body mass requires 33 watts of power to drive the animal at 30 km/h. Assuming a 25% muscle efficiency [24], the mechanical work done would be about 8 watts-kg @ 30 km/h. This is approximately equal to the 7 watts-kg @ 30 km/h calculated by examining data published in Cavagna and colleagues [20] for dog locomotion.

The energy conversion of an electro-mechanical system is roughly that of human muscle. Using data from Poulakakis and colleagues [21] we computed that the Scout II robot has a 33% energy conversion efficiency from electrical power to mechanical power. Using data from various sources, we computed that the theoretical maximum efficiency of a combined electronics + gear train+ motor should be about 50-60%. However, the efficiency of an electric motor depends significantly on its operating region. The Poulakakis data seem plausible and better than expected.

Specific resistance measures the effort that a mode of transportation uses to move through the environment [25]. It is expressed as the ratio of power used versus the power needed to move the same mass straight up under the force of gravity. The energetic cost of transport is a similar measure using energy instead of power. For our analysis, the terms can be used interchangeably.

The biggest advance in robot energetics will be in the reduction of specific resistance of the robotic cheetah closer to that of animal data. Poulakakis and colleagues [21] states that his robot, the most efficient galloping machine to date, has a specific resistance of about ϵ = 0.47. In contrast, the cheetah, from the data above, has a specific resistance of about ϵ = .098. If the Cheetah is used as an existence proof, it is possible to reduce the specific resistance in a running cheetah robot by a factor of almost 5 times over what has previously been reported.

Why is energy important? If we achieved cheetah like specific resistance and had energy conversion at a conservative 20%, then a 25 kg running cheetah-robot would use less than 1000 watts and be able to cover 30 km in 1 hour. Further, given an energy density of lithium batteries of 200 $watt \cdot h^{-1} \cdot kg^{-1}$, we would need only 5 kg of batteries to accomplish this task. The state of the art, however, dictates the use of a 25 kg battery pack. Essentially the entire robot would be a battery, with no room for actuators or mechanics.

Another way of looking at this is that for a fixed power budget, a 2 times increase in efficiency translates to a 2 times increase in speed of the robot (under the condition that aerodynamic effects are not taken into account). The origin of locomotor efficiency must be in the mechanics of the limb design.

III. Cheetah Locomotion

In this section we review specific properties of cheetah locomotion.

A. Physical Characteristics

The cheetah runs with a rotary gallop where foot falls touch the ground in a pattern that rotates from the fore limbs around the body to the hind limbs. The fore limbs touch down first when the cheetah strikes the ground from flight phase. In contrast, the horse runs with a transverse gallop and its *hind limbs* touch down first [27].

The rotary gallop is also used by dogs and it is from dog data that we can get an idea of the ground forces necessary to achieve high speeds. When galloping at nearly 10 meters per sec, the dog *fore limbs* exert up to 2.5 times the animal's

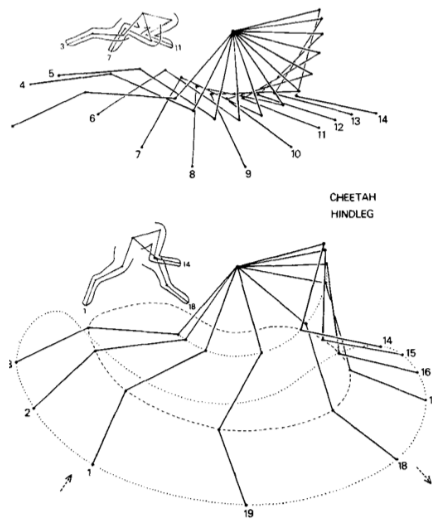

Fig. 1. Movement of Cheetah Hind Limb during high speed galloping, adapted from [26].

TABLE I
APPROXIMATE CHEETAH LIMB JOINT PROPERTIES

Joint	Min Ang (deg)	Max Ang (deg)	Ang Range (deg)	Max Ang Vel (deg/s)
Spine-Hip	-131	14	145	2206
Knee	-138	-32	104	2300
Ankle	127	32	95	1763

body weight (BW) for each limb when hitting the ground and the *hind limbs* about 1.5 BW. Human runners also exert a maximum of about 2.5 BW when running and the vertical ground reaction force does not seem to increase with running speed. It is thought that the dog fore limb more closely mimics the dynamics of a human runner, in particular with regard to energy storage and return. While we cannot be certain, at speeds of about 10 m/s, the maximum force exerted by a cheetah maybe 2.5 BW for the front limbs, and 1.5 BW for the hind limbs, similar to the dog.

Energy storage during deceleration and return during acceleration is a key feature of both human running, as well as quadrupedal galloping. In animals, energy storage and return is primarily concentrated more at the distal (away from the body) segments of the animal limb and less so in the proximal segments. Further, much more energy storage in the dog occurs in the fore limbs than the hind limbs. The hip has no measurable energy storage in stance during gallop. There may be some energy storage during swing phase, as suggest by Alexander [14] but that would have a small impact on the overall efficiency of locomotion.

In Table I, we have extracted the limb movement from Fig 1. The angular positions for a stride are plotted in Fig. 2 and velocity in Fig. 3. From our calculations, the joints of the cheetah reach a peak at a velocity of about 2000 degrees per second, see Fig. 3.

The cheetah is unique in the use of extreme spinal flex-

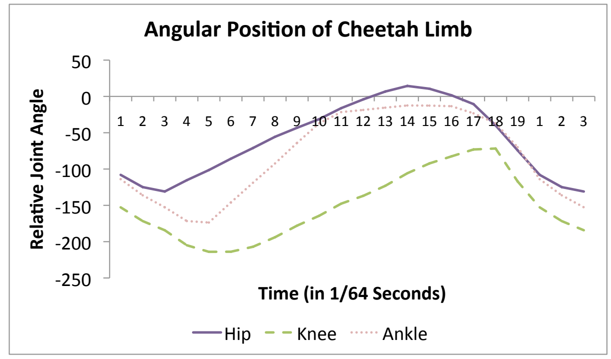

Fig. 2. Angular Position of Cheetah Hind Limb during gallop. Extract from [26]. Stance is at times 18, 19 and 1.

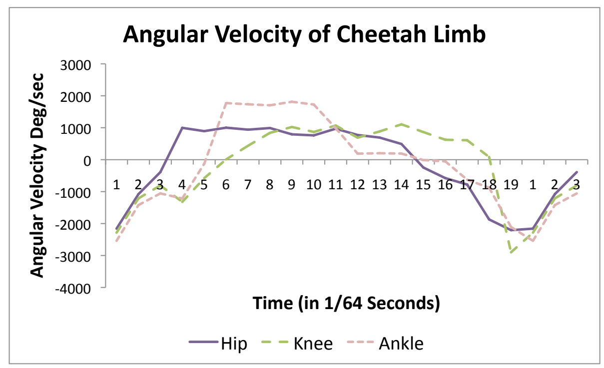

Fig. 3. Angular Velocity of Cheetah Hind Limb during gallop. Extract from [26]. Stance is at times 18, 19 and 1.

ion during locomotion. From observations of cheetah videos available on the web, it appears that the spine flexes about 90 degrees during running. Hence the range of movement documented by Hildebrand and Hurley [26] for the hip represents the *combined* action of the hip plus spine, because as the spine flexes, the hip also rotates.

Note that in Table I, the spine-hip angle has a range of 145 degrees, and it appears that the spine is contributing about 90 degrees to this figure, or about two-thirds of the range of motion. Since the spine-hip torque is approximately equal to the hip-femur torque, we can conclude that up to two-thirds of the work a the spine-hip joint may be contributed by the spine.

Looking at Figures 3, we see that an actuator system must be capable of generating extremely high velocities to achieve cheetah like speed. As noted in the introduction, robots that run at high velocities have made use of pneumatic, gas powered cylinders. Air actuated muscles are difficult to control precisely. We are aware of many attempts to build air powered robots with precise control which never lived up to their promise. Unfortunately, negative results are often not published in the robotics literature.

How can we achieve both high power during stance phase as well as precise movements needed for slower, more controlled maneuvers?

IV. Cheetah Hind Limb Design

In this section, we give an overview of a cheetah Hind Limb design using novel actuator concepts.

The key design goal of the Cheetah hind limb was to create a limb that could generate 1.5 times the estimated body weight of the complete cheetah in the vertical direction. This would match the force produced by the dog hind limb during gallop. At the same time, the robot should be capable of walking in a precise manner.

Pneumatic actuators have the advantage of achieving relatively high force in a compact space. Pneumatic actuator and valves are readily available that can operate on up to 1700 kPa (250 psi). For example, an off-the-shelf commercial grade cylinder with a 8 cm stroke is capable of providing 78 kg weighs 200 g.

Pneumatic cylinders can also be used as nonlinear springs to absorb shock, and to recover energy. During running, it is estimated that well over 50% of power in leg movements comes from strain energy released by tendons and muscles.

On the other hand, electric motors are excellent for precise control and are capable of very high power densities. However, high gear reduction, necessary for efficient motor operation leads to non-backdrivable, non-compliant systems. Pratt invented the Series Elastic Actuator (SEA) which can restore compliance to motor systems [28]. However, we are not aware of studies showing the energy storage and release capability of SEAs.

A. Hybrid actuator concept

Our approach is to use electric motors for fine positioning control and then to inject ballistically energy when needed for galloping and trotting tasks via pneumatic actuators. We assume that bursts of power need not be precisely controlled. This conjecture is supported by strong empirical evidence (1) we note that in non-articulated robot legs, such as Raibert's hoppers, fine control of force was not necessary for running and (2) the ground reaction force component along the gravity vector is a relatively simple, unimodal curve, (3) the contact time is too short for closed loop circuits to be used for fine force control in the limb of animals. The burst of energy for running is ballistic and would therefore be a good candidate for a pneumatic actuator.

How can we combine the force output of two types of actuators non-destructively? If we combine two actuator outputs together, no problem exists if the devices are identical and are driven by the same control commands.

If we have two motors with *different* dynamical characteristics we run the risk of one motor applying torque in one direction and another actuator applying torque in the opposite direction. In this case one actuator is doing positive work and the actuator absorbing work is said to be doing negative work. Under these circumstances, the system is less efficient and there is the risk that one actuator may damage the other.

We note that this problem is similar to the electrical problem of combining two power supplies of different voltages. If two supplies of differing voltages are connected together directly, the higher voltage supply will end up causing a current flow into the lower voltage supply. Electrically, we would use a diode to prevent undesirable current flows. A *tendon* is the mechanical equivalent of a diode, allowing force in one direction but not the other.

Fig. 4 shows our tendon architecture used for both knee and ankle. Here we show three modes of operation (Fig 4, A-C). In a typical mode of operation the pneumatic cylinder can provide a biasing force which assists the electric motor in its operation. When necessary, a burst of air is injected into the pneumatic muscle to generate an impulse needed for jumping. When a collision occurs, the pneumatic cylinder can be used as an energy absorbing spring.

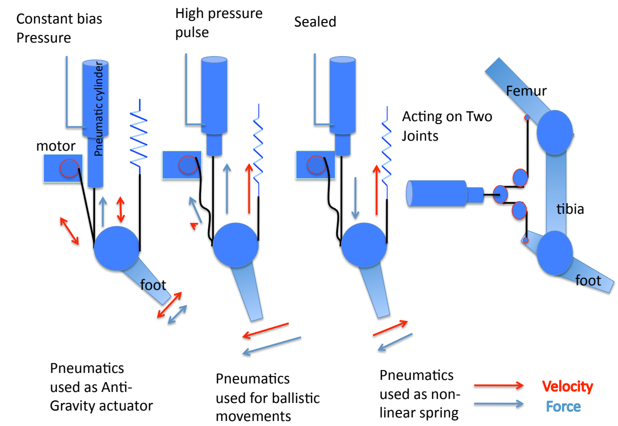

Fig. 4. Hybrid actuator concept. From left: (A) Pneumatics can be used as anti-gravity actuators providing a bias force against gravity. This reduces torque loading on an electric motor. (B) When ballistic movements are needed, the pneumatics can generate high forces. (C) Upon landing, the actuators are used as non-linear springs. (D) In addition, biarticular actuators, acting about two joints are the primary actuators for ballistic movements in the leg. Two biarticular actuators, mimicking the gastrocnemius and the rectus femoris are used.

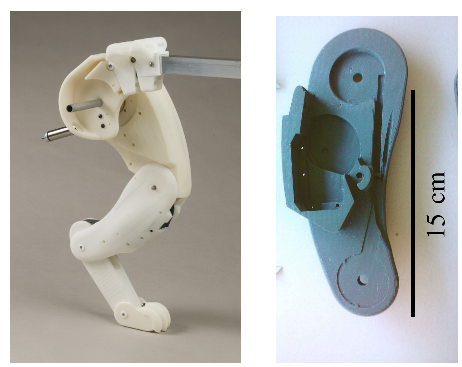

Fig. 5. Construction of the robot leg. The leg is constructed from ABS plastic. Using techniques developed over several years, we create the body parts by tightly embedding components inside the a clam shell. Each clam shell itself has a sparse-built, hollow interior. This process allows us to create a aesthetically appealing, but very strong and lightweight structure for the robot. The material for the tibia (the middle segment, both halves, weights only 127 grams-force, yet can withstand the large forces placed on it by the Pneumatic muscles.)

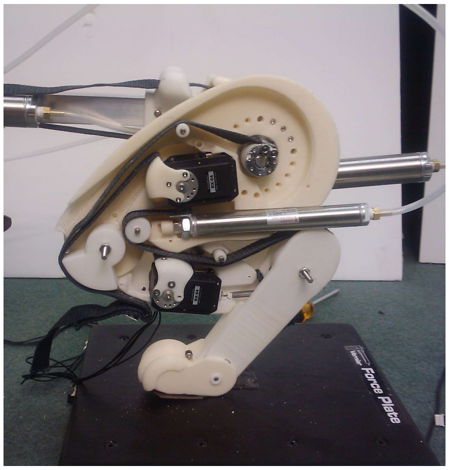

Fig. 6. Realization of the Cheetah Robotic Limb

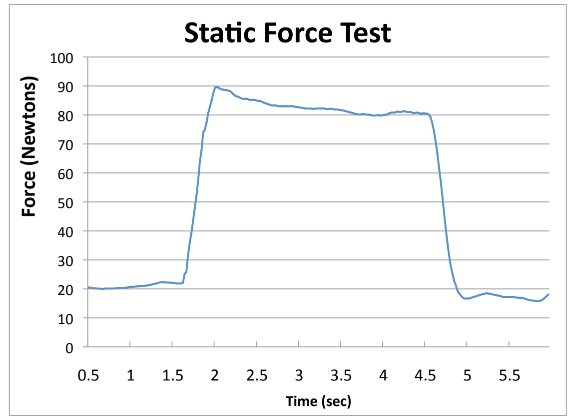

Fig. 7. Static force test. The robot is held down by an experimenter and 130 psi is applied. The weight of the robot is about 22 Newtons.

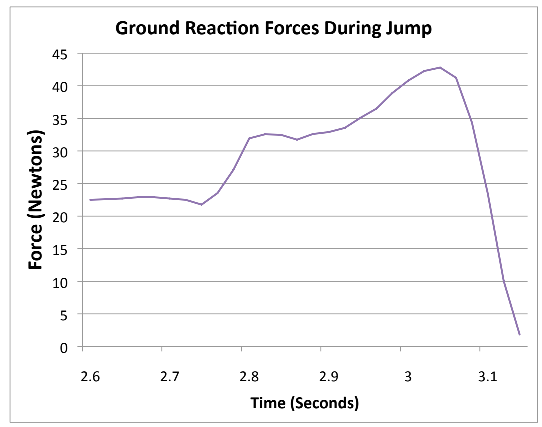

Fig. 8. Ground Reaction Force during jump.

B. Use of Biarticular Muscles

We incorporate biarticular muscles in our robot driven by pneumatic actuators, Fig. 4(d). Biarticular muscles are thought to have a critical role in explosive movements. Further, Seyfarth has pointed out that biarticular muscles help to stabilize the overall structure of an articulated leg [10].

In previous work, we have explored the energy transfer mechanism of biarticular muscles in a robot [29], [30]. In that work, we were able to demonstrate power transfer from hip to ankle in a robot using biarticular muscles.

C. Robot Construction

The robot was constructed in ABS plastic using a Dimension 3d printer. The 3-d printer allows parts to be built with a sparse matrix of support internally. Through years of experimentation we have perfected a clam shell approach which results in an anesthetically pleasing, yet strong and lightweight body design, see Fig. 5. The total weight of a single robot leg, its hip and the support fixture is 22 Newtons. The weight of the leg alone is about 17 Newtons.

D. Tendon Architecture

Fig. 6 shows a cutaway view of the robot leg showing the position of pneumatic cylinders as well as electric actuators. Note that in this design, we do not use a traditional "agonist-antagonist" architecture. The agonist "muscle" is simply a spring (see Fig. 4). Spring use simplifies the architecture of the limb and reduces limb weight. However, we do sacrifice some ability to control selectively the compliance of each limb joint, and we have a minor reduction in the efficiency of the extensor actuators.

E. Novelty

Khatib and colleagues have also developed a hybrid actuator which combines pneumatics and electric motors [31]. In their approach, McKibben muscles are used in agonist-antagonist pairs. Pneumatic actuators and electric motors act in parallel.

The robot Mowgli [32] uses uses *biarticular* muscles as we do.

The work presented here is designed for legged locomotion, both walking and running and differs from the above work in a number of ways. (1) Pneumatics are used ballistically. We do not attempt to regulate the exact force at of the pneumatic actuator. (2) The pneumatics are very low bandwidth. We anticipate turning valves on and off at a rate of approximately 3Hz. This is important as it should significantly reduce energy loses versus a system which tries to control tightly torque. (3) We include biarticular muscles. These muscles are important for explosive movements in humans. We incorporate those as well. (4) Pneumatics and electric motors are used for extension, while flexion is by use of a passive spring. (5) We use high gear reduction electric motors, which can be very efficient. Our tendon system automatically decouples the electric motor from the pneumatic actuators when the pneumatic actuators activate with high force.

V. Results

In initial tests, we verified that we could achieve fine motor movement using electric motors alone. This was done by applying as simple pattern signal to the biarticular actuators. As expected, the robot moved precisely. Of more interest were tests using the pneumatic muscles. These are of two types: Static tests and dynamic tests.

A. Static Force Test

In our first experiment we determined the maximum force that the robot could exert while pushing against gravity. In this test, an experimenter held the robot down while the pneumatic biarticular muscles or the electrical motors were activated. The robot was then allowed to rise slowly.

In this test 130 psi air supply was used. The actuators are rated up to 250 psi, but because of limitation of our current solenoid valves, we could only apply about 50% of the maximum power. Ground Reaction force data were collected using a Vernier Corporation Force plate, MDL FP-BTA.

The data was sampled at 50 Hz and was averaged locally using a window size of 10 samples. In this test, our leg force peaked at about 90 Newtons, see Fig. 7. Future experiments will use an air supplies of about 200+ psi. This increased pressure should results in 50% more force, about 135 Newtons.

In a separate test we performed the same experiment using the electric motors alone. In that experiment our peak force was about 40 Newtons.

B. Dynamic Test

A second test was performed to determine the performance of the leg in a kicking maneuver. The leg was placed in a fully flex (crouched) position. This represents a worst case starting position. The leg was then energized using biarticular muscles alone. In this case, we measured both the ground reaction force in the direction along the gravity vector and recorded the resulting jump using a digital video recorder. The ground reaction forces were again sampled at 50 Hz and filtered using a running average over 3 samples, to capture better the dynamics. Video frames were recorded at 30 frames per second and were digitized by hand.

In Fig. 8 we see the ground reaction forces. The peak force was about 43 Newtons. In Fig. 9 we see the digitized jump of the robot. The robot leapt off the platform and kicked backward very swiftly, see Fig. 10. The joint velocities, as measured from the video recording shown in Fig. 9 are plotted in Fig. 11. The peak velocity was about 1050 degrees per second for the hip, 900 degrees per second for the knee and 1600 degrees per second for the ankle. This compares to 2200 for the hip-spine for the real cheetah, 2300 for the knee and 1700 for the ankle. Thus, our results are about half of our estimated cheetah performance for the knee and hip-spine. See section III-A. In the current test, the robot leg left the ground near from 7 as it was achieving peak force.

We note that the leg did achieve about 2 times the weight of the leg assembly when jumping. However, the full robot will weigh an estimated 100 Newtons. Our current robot leg can

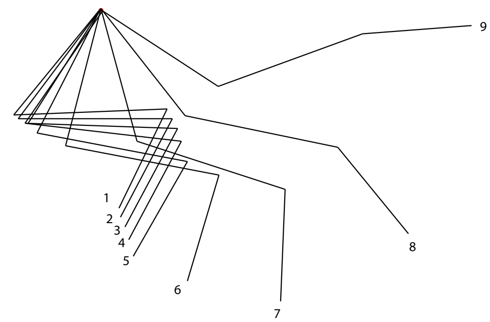

Fig. 9. Digitized video recording of movement of leg during the dynamic kick test. The stick figures above represent 9 video frames 33 ms apart. All data is with respect to the hip. At frame 7, the leg is still on the ground. At frame 8 the leg is airborne.

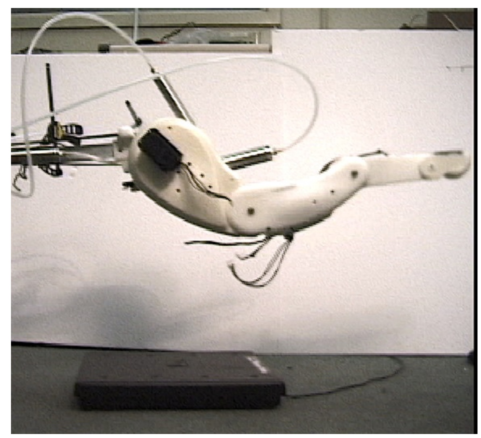

Fig. 10. Robot leg airborne in mid kick

achieve 70 Newtons of force using a lower pressure supply. We may achieve higher static (and dynamic) forces with a high pressure power supply. However, we also note that in the case where the leg collides with the ground, much of that energy can be absorbed by pneumatic springs. A large portion of the ground reaction forces are due to elastic elements which we have not tested here. It is likely that even give our current design, if we dropped the robot leg, we would achieve much higher ground reaction forces.

VI. Conclusion and Future Work

The main results of this paper are (1) using published data, we were able to determine key figures for the design of a robot cheetah: Range of motion of the joint, the maximum joint speed, power usage and needed ground reaction forces. (2) We created an innovative hybrid actuator concept which uses both electric motors as well as pneumatic actuators to achieve both fine control as well as jumping. (3) We measured the force output of the current actuator system. In static tests we could achieve about .7 BW of the entire robot or 3 BW of the test leg alone. We made the argument that with increased air pressure we should be able to achieve near 1.35 BW of

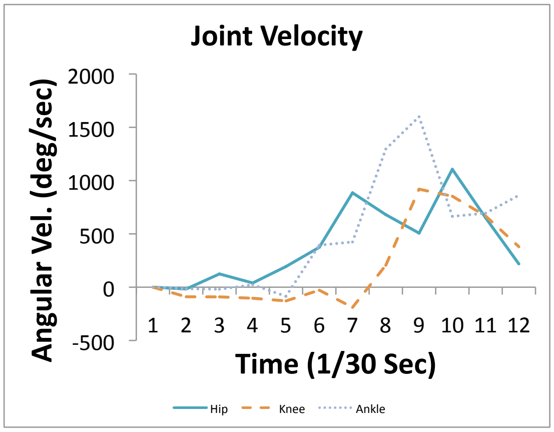

Fig. 11. Kick test. Angular velocity.

force. (4) We achieved relatively high velocities in the kick test approaching about half the speed of the real cheetah.

We note that the kick test is a fundamentally different pattern than that used by a leg during locomotion, but we feel that our results show that we are achieving performance approaching that of the cheetah, and with sufficient engineering effort we should be able to match the cheetah's performance.

In future work we will use higher air pressure to increase the force output of the leg as well as using pneumatic muscles as nonlinear springs. After the second leg is completed, we will be able to test the "galloping" behavior of the real limb system of the robot.

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